

Off-shell $N = (4, 4)$ supersymmetry for new $(2, 2)$ vector multiplets

M. Göteman^a, U. Lindström^a, M. Roček^b, and I. Ryb^{ab}

*a: Theoretical Physics,
Department Physics and Astronomy,
Uppsala University,
Box 803, SE-751 08 Uppsala, Sweden*

*b: C.N. Yang Institute for Theoretical Physics,
Stony Brook University,
Stony Brook, NY 11794-3840, USA*

Abstract

We discuss the conditions for extra supersymmetry of the $N = (2, 2)$ supersymmetric vector multiplets described in arXiv:0705.3201 [hep-th] and in arXiv:0808.1535 [hep-th]. We find $(4, 4)$ supersymmetry for the semichiral vector multiplet but not for the Large Vector Multiplet.

Contents

1	Introduction	1
2	Review of the new gauge multiplets	2
2.1	The semichiral vector multiplet	3
2.2	The Large Vector Multiplet	4
3	$N=(4,4)$ susy for the semichiral multiplet	5
3.1	The abelian semichiral multiplet	5
3.2	The nonabelian semichiral vector multiplet	6
4	No $N=(4,4)$ susy for the Large Vector Multiplet	10
5	Conclusions	11
A	Hermiticity conventions	12

1 Introduction

In a recent paper [1], we investigated under what conditions a manifest $N=(2,2)$ sigma model written entirely in terms of left and right semichiral superfields [2] admits $N=(4,4)$ supersymmetry [3, 4]. Here we continue to investigate extended supersymmetry in generalized Kähler geometry focusing on the $(2,2)$ vector multiplets introduced in [5, 6] which were used to gauge isometries and to discuss T-duality in [7]. See also the related papers [8] and [9].

We find an off-shell $(4,4)$ algebra for semichiral vector multiplet and present it both at the level of field-strengths and at the level of gauge (pre-)potentials. The algebra closes up to gauge transformations which we calculate.

For the Large Vector Multiplet (LVM) the situation is different. There cannot exist an off-shell $(4,4)$ supersymmetry for a single LVM. Instead, we find a pseudo-supersymmetry in the abelian LVM, and comment on obstacles in the nonabelian case.

We have organized the paper as follows: In section 2 we recapitulate the basic definitions and properties of the $N=(2,2)$ vector multiplets. In section 3 we present the $N=(4,4)$ supersymmetry for the semichiral vector multiplet and in section 4 we discuss (twisted) supersymmetry for the Large Vector Multiplet. Conclusions and outlook are contained in the last section.

2 Review of the new gauge multiplets

In this section we review material needed in the rest of the paper. The most general manifest $N = (2, 2)$ supersymmetric sigma model is described by a generalized Kähler potential¹ which is a function of chiral, twisted chiral and semichiral fields [10],

$$K = K(\phi, \bar{\phi}, \chi, \bar{\chi}, \mathbb{X}_L, \bar{\mathbb{X}}_L, \mathbb{X}_R, \bar{\mathbb{X}}_R) , \quad (2.1)$$

where the constraints on the $N = (2, 2)$ superfields are

$$\bar{\mathbb{D}}_{\pm}\phi = \bar{\mathbb{D}}_{+}\chi = \mathbb{D}_{-}\chi = \bar{\mathbb{D}}_{+}\mathbb{X}_L = \bar{\mathbb{D}}_{-}\mathbb{X}_R = 0 . \quad (2.2)$$

The target space of this $N = (2, 2)$ sigma model has been shown [3] to admit a bihermitian structure which is equivalent to a generalized Kähler structure [11]. The simplest isometries act on purely Kähler submanifolds of the generalized Kähler geometry, that is only on the chiral superfields ϕ or the twisted chiral superfields χ ;

$$K_{\phi} = K(\phi + \bar{\phi}, \dots) , \quad K_{\chi} = K(\chi + \bar{\chi}, \dots) . \quad (2.3)$$

These potentials have a rigid isometry, $\delta\phi = i\lambda$ for K_{ϕ} and $\delta\chi = i\lambda$ for K_{χ} with λ a real constant parameter. When gauged, however, the local parameter must respect the chirality properties of the superfields,

$$\delta_g\phi = i\Lambda \quad \Rightarrow \quad \bar{\mathbb{D}}_{\pm}\Lambda = 0 ; \quad \delta_g\chi = i\Lambda \quad \Rightarrow \quad \bar{\mathbb{D}}_{+}\Lambda = \mathbb{D}_{-}\Lambda = 0 . \quad (2.4)$$

To ensure the invariance of the Lagrange densities (2.3) under the local transformations, we introduce the appropriate vector multiplets. These give the well known transformation properties for the usual (un)twisted vector multiplets:

$$\begin{aligned} \delta_g V^{\phi} &= i(\bar{\Lambda} - \Lambda) \quad \Rightarrow \quad \delta_g(\phi + \bar{\phi} + V^{\phi}) = 0 , \\ \delta_g V^{\chi} &= i(\bar{\tilde{\Lambda}} - \tilde{\Lambda}) \quad \Rightarrow \quad \delta_g(\chi + \bar{\chi} + V^{\chi}) = 0 . \end{aligned} \quad (2.5)$$

The field-strength is twisted chiral for V^{ϕ} and chiral for V^{χ} :

$$\tilde{W} = \bar{\mathbb{D}}_{+}\mathbb{D}_{-}V^{\phi} , \quad W = \bar{\mathbb{D}}_{+}\bar{\mathbb{D}}_{-}V^{\chi} . \quad (2.6)$$

Isometries that involve other combinations of the fields in (2.1) have been suggested in [12] and recently discussed in [5, 7]. We now describe the new multiplets that can be used to gauge these isometries.

¹The description holds locally and away from irregular points.

2.1 The semichiral vector multiplet

An example of a sigma model with a rigid symmetry that acts on semichiral superfields is given by a potential of the form

$$K = K(\phi, \bar{\phi}, \chi, \bar{\chi}, \mathbb{X}_L + \bar{\mathbb{X}}_L, \mathbb{X}_R + \bar{\mathbb{X}}_R, i(\mathbb{X}_L - \bar{\mathbb{X}}_L - \mathbb{X}_R + \bar{\mathbb{X}}_R)) . \quad (2.7)$$

The rigid isometry is $\delta \mathbb{X}_L = \delta \mathbb{X}_R = i\lambda$. The gauging of such an isometry requires the chirality properties of the parameters

$$\delta_g \mathbb{X}_L = i\Lambda_L \Rightarrow \bar{\mathbb{D}}_+ \Lambda_L = 0 , \quad \delta_g \mathbb{X}_R = i\Lambda_R \Rightarrow \bar{\mathbb{D}}_- \Lambda_R = 0 . \quad (2.8)$$

A vector multiplet corresponding to this isometry was introduced in [5, 8]. This semichiral vector multiplet is described by three real potentials $(\mathbb{V}^L, \mathbb{V}^R, \mathbb{V}')$ with gauge transformations

$$\delta_g \mathbb{V}^L = i(\bar{\Lambda}_L - \Lambda_L) , \quad \delta_g \mathbb{V}^R = i(\bar{\Lambda}_R - \Lambda_R) , \quad \delta_g \mathbb{V}' = (-\Lambda_L - \bar{\Lambda}_L + \Lambda_R + \bar{\Lambda}_R) , \quad (2.9)$$

which implies that

$$\begin{aligned} \delta_g(\mathbb{X}_L + \bar{\mathbb{X}}_L + \mathbb{V}^L) &= 0 , \quad \delta_g(\mathbb{X}_R + \bar{\mathbb{X}}_R + \mathbb{V}^R) = 0 \\ \delta_g(\mathbb{X}_L - \bar{\mathbb{X}}_L - \mathbb{X}_R + \bar{\mathbb{X}}_R + i\mathbb{V}') &= 0 . \end{aligned} \quad (2.10)$$

We construct field-strengths from complex combinations of the real potentials:

$$\begin{aligned} \mathbb{V}_\phi &= \frac{1}{2} (i\mathbb{V}' + \mathbb{V}^L - \mathbb{V}^R) , \quad \delta_g \mathbb{V}_\phi = i(\Lambda_R - \Lambda_L) , \\ \mathbb{V}_\chi &= \frac{1}{2} (i\mathbb{V}' + \mathbb{V}^L + \mathbb{V}^R) , \quad \delta_g \mathbb{V}_\chi = i(\bar{\Lambda}_R - \Lambda_L) . \end{aligned} \quad (2.11)$$

These potentials satisfy the reality condition

$$\bar{\mathbb{V}}_\phi - \mathbb{V}_\phi = \bar{\mathbb{V}}_\chi - \mathbb{V}_\chi . \quad (2.12)$$

The gauge invariant field-strengths of the semichiral vector multiplet are chiral and twisted chiral superfields,

$$\mathbb{F} = i\bar{\mathbb{D}}_+ \bar{\mathbb{D}}_- \mathbb{V}_\phi , \quad \tilde{\mathbb{F}} = i\bar{\mathbb{D}}_+ \mathbb{D}_- \mathbb{V}_\chi , \quad \bar{\mathbb{F}} = i\mathbb{D}_+ \bar{\mathbb{D}}_- \bar{\mathbb{V}}_\phi , \quad \tilde{\bar{\mathbb{F}}} = i\mathbb{D}_+ \bar{\mathbb{D}}_- \bar{\mathbb{V}}_\chi . \quad (2.13)$$

In the left semichiral representation [5], [6], we can introduce covariant derivatives as

$$\begin{aligned} \bar{\nabla}_+ &= \bar{\mathbb{D}}_+ \\ \bar{\nabla}_- &= e^{-\mathbb{V}_\phi} \bar{\mathbb{D}}_- e^{\mathbb{V}_\phi} \\ \nabla_+ &= e^{-\mathbb{V}_\phi} e^{\bar{\mathbb{V}}_\chi} \mathbb{D}_+ e^{-\bar{\mathbb{V}}_\chi} e^{\mathbb{V}_\phi} = e^{-\bar{\mathbb{V}}_\chi} e^{-\bar{\mathbb{V}}_\phi} \mathbb{D}_+ e^{\bar{\mathbb{V}}_\phi} e^{\bar{\mathbb{V}}_\chi} = e^{-\mathbb{V}^L} \mathbb{D}_+ e^{\mathbb{V}^L} \\ \nabla_- &= e^{-\bar{\mathbb{V}}_\chi} \mathbb{D}_- e^{\bar{\mathbb{V}}_\chi} . \end{aligned} \quad (2.14)$$

For the abelian case, this gives the following set of connections:

$$\begin{aligned}
\bar{\Gamma}_+ &= 0 \\
\bar{\Gamma}_- &= \bar{\mathbb{D}}_- \mathbb{V}_\phi \\
\Gamma_- &= \mathbb{D}_- \mathbb{V}_\chi \\
\Gamma_+ &= \mathbb{D}_+ \mathbb{V}^L = \mathbb{D}_+ (\mathbb{V}_\phi + \bar{\mathbb{V}}_\chi) = \mathbb{D}_+ (\bar{\mathbb{V}}_\phi + \mathbb{V}_\chi) .
\end{aligned} \tag{2.15}$$

Correspondingly, the field-strengths read

$$\begin{aligned}
\mathbb{F} &= i\{\bar{\nabla}_+, \bar{\nabla}_-\} = i\bar{\mathbb{D}}_+ \bar{\Gamma}_- \\
\tilde{\mathbb{F}} &= i\{\bar{\nabla}_+, \nabla_-\} = i\bar{\mathbb{D}}_+ \Gamma_- \\
\bar{\mathbb{F}} &= -i\{\nabla_+, \nabla_-\} = -i(\mathbb{D}_+ \Gamma_- + \mathbb{D}_- \Gamma_+) \\
\tilde{\bar{\mathbb{F}}} &= -i\{\nabla_+, \bar{\nabla}_-\} = -i(\mathbb{D}_+ \bar{\Gamma}_- + \bar{\mathbb{D}}_- \Gamma_+) ,
\end{aligned} \tag{2.16}$$

where the last equalities again refer to the abelian case.

2.2 The Large Vector Multiplet

An example of a sigma model with a rigid symmetry $\delta\phi = \delta\chi = i\lambda$ that acts simultaneously on chiral and twisted chiral superfields is given by a potential of the form

$$K = K(\phi + \bar{\phi}, \chi + \bar{\chi}, i(\phi - \bar{\phi} - \chi + \bar{\chi}), \mathbb{X}_L, \bar{\mathbb{X}}_L, \mathbb{X}_R, \bar{\mathbb{X}}_R) . \tag{2.17}$$

This isometry is gauged by the Large Vector Multiplet (LVM) with three real potentials (V^ϕ, V^χ, V') , where in addition to the gauge transformations of V^ϕ and V^χ in (2.5), the potential V' transforms as

$$\delta_g V' = (-\Lambda - \bar{\Lambda} + \tilde{\Lambda} + \bar{\tilde{\Lambda}}) . \tag{2.18}$$

As for the semichiral multiplet, we introduce complex combinations

$$\begin{aligned}
V_L &= \frac{1}{2}(-V' + i(V^\phi - V^\chi)) \Rightarrow \delta_g V_L = \Lambda - \tilde{\Lambda} , \\
V_R &= \frac{1}{2}(-V' + i(V^\phi + V^\chi)) \Rightarrow \delta_g V_R = \Lambda - \bar{\tilde{\Lambda}} ,
\end{aligned} \tag{2.19}$$

subject to the reality condition

$$V_L + \bar{V}_L = V_R + \bar{V}_R . \tag{2.20}$$

We construct gauge invariant field-strengths for the Large Vector Multiplet as

$$\mathbb{G}_+ = \bar{\mathbb{D}}_+ V_L , \quad \mathbb{G}_- = \bar{\mathbb{D}}_- V_R , \tag{2.21}$$

and their complex conjugate. The field-strengths for the LVM are thus semichiral spinors.

3 $N=(4,4)$ susy for the semichiral multiplet

For simplicity we begin this section with a discussion of the abelian case.

3.1 The abelian semichiral multiplet

It is well-known that a chiral and a twisted chiral superfield allow $(4,4)$ supersymmetry [3].² It follows that the four field-strengths (2.16) transform under $(4,4)$ supersymmetry according to

$$\begin{aligned}\delta_Q \mathbb{F} &= \epsilon^+ \bar{\mathbb{D}}_+ \tilde{\mathbb{F}} + \epsilon^- \bar{\mathbb{D}}_- \tilde{\mathbb{F}} \\ \delta_Q \bar{\mathbb{F}} &= \bar{\epsilon}^+ \mathbb{D}_+ \tilde{\mathbb{F}} + \bar{\epsilon}^- \mathbb{D}_- \tilde{\mathbb{F}} \\ \delta_Q \tilde{\mathbb{F}} &= -\epsilon^+ \bar{\mathbb{D}}_+ \bar{\mathbb{F}} - \bar{\epsilon}^- \mathbb{D}_- \bar{\mathbb{F}} \\ \delta_Q \tilde{\bar{\mathbb{F}}} &= -\bar{\epsilon}^+ \mathbb{D}_+ \bar{\mathbb{F}} - \epsilon^- \bar{\mathbb{D}}_- \bar{\mathbb{F}} .\end{aligned}\tag{3.1}$$

This gives an algebra that closes off-shell;

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]\mathbb{F} = \xi^+ \partial_+ \mathbb{F} + \xi^- \partial_- \mathbb{F} ,\tag{3.2}$$

where we labeled products of supersymmetry parameters as

$$\xi^\pm = i\epsilon_{[2}^\pm \bar{\epsilon}_{1]}^\pm ,\tag{3.3}$$

and the supersymmetry algebra is $\{\bar{\mathbb{D}}_\pm, \mathbb{D}_\pm\} = i\partial_\pm$. The corresponding transformations on the potentials \mathbb{V}_ϕ and \mathbb{V}_χ can then be deduced using this ansatz together with the constraint on the imaginary part (2.12),

$$\begin{aligned}\delta_Q \mathbb{V}_\phi &= -\epsilon^- \bar{\mathbb{D}}_- \mathbb{V}_\chi - \epsilon^+ \mathbb{D}_+ \bar{\mathbb{V}}_\chi + \bar{\epsilon}^- \bar{\mathbb{D}}_- \mathbb{V}_\phi - \bar{\epsilon}^+ \bar{\mathbb{D}}_+ \mathbb{V}_\phi \\ \delta_Q \mathbb{V}_\chi &= \epsilon^+ \mathbb{D}_+ \bar{\mathbb{V}}_\phi + \bar{\epsilon}^- \bar{\mathbb{D}}_- \mathbb{V}_\phi - \epsilon^- \bar{\mathbb{D}}_- \mathbb{V}_\chi + \bar{\epsilon}^+ \bar{\mathbb{D}}_+ \mathbb{V}_\chi .\end{aligned}\tag{3.4}$$

The supersymmetry algebra closes on the potentials up to gauge transformations according to:

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] \begin{pmatrix} \mathbb{V}_\phi \\ \mathbb{V}_\chi \end{pmatrix} = [\xi^- \partial_- + \xi^+ \partial_+] \begin{pmatrix} \mathbb{V}_\phi \\ \mathbb{V}_\chi \end{pmatrix} + \begin{pmatrix} \Lambda_R - \Lambda_L + 2\tilde{\alpha}\tilde{\mathbb{F}} + 2\tilde{\bar{\alpha}}\tilde{\bar{\mathbb{F}}} \\ \bar{\Lambda}_R - \Lambda_L + 2\alpha\mathbb{F} + 2\bar{\alpha}\bar{\mathbb{F}} \end{pmatrix} ,\tag{3.5}$$

²Various $(4,4)$ models that involve chiral and twisted chiral superfields have been discussed in [13]. The application to the present situation will be described in [14].

where

$$\begin{aligned}
\alpha &= i\epsilon_{[2}^+\epsilon_{1]}^-, & \tilde{\alpha} &= i\epsilon_{[2}^+\tilde{\epsilon}_{1]}^- \\
\Lambda_L &= -i\xi^{\pm\pm} \bar{\mathbb{D}}_+\mathbb{D}_+(\bar{\mathbb{V}}_\chi + \mathbb{V}_\phi) \\
\Lambda_R &= -i\xi^{\pm\pm} \bar{\mathbb{D}}_-\mathbb{D}_-(\mathbb{V}_\chi - \mathbb{V}_\phi) .
\end{aligned} \tag{3.6}$$

Note that the field-strengths appearing in (3.5) have the correct chirality to serve as gauge transformations.

Substituting the transformation laws for the potentials in (3.4) in equation (2.15) we find, in the left semichiral representation

$$\begin{aligned}
\delta_Q \bar{\Gamma}_+ &= 0 \\
\delta_Q \Gamma_+ &= i\epsilon^-\bar{\mathbb{F}} - i\bar{\epsilon}^-\tilde{\mathbb{F}} + \epsilon^-\mathbb{D}_+\Gamma_- - \bar{\epsilon}^-\mathbb{D}_+\bar{\Gamma}_- \\
\delta_Q \bar{\Gamma}_- &= i\epsilon^+\tilde{\mathbb{F}} + i\bar{\epsilon}^+\mathbb{F} + \epsilon^-\bar{\mathbb{D}}_-\Gamma_- \\
\delta_Q \Gamma_- &= -i\bar{\epsilon}^+\tilde{\mathbb{F}} - i\epsilon^+\bar{\mathbb{F}} - \bar{\epsilon}^-\mathbb{D}_-\bar{\Gamma}_- .
\end{aligned} \tag{3.7}$$

We go to a real representation using the shift $\delta_g \Gamma = -iD(\Xi)$ where

$$\Xi = i(\epsilon^-\Gamma_- - \bar{\epsilon}^-\bar{\Gamma}_-) . \tag{3.8}$$

This puts all Γ transformations on equal footing:

$$\begin{aligned}
\delta_Q \bar{\Gamma}_+ &= i\epsilon^-\tilde{\mathbb{F}} - i\bar{\epsilon}^-\mathbb{F} \\
\delta_Q \Gamma_+ &= -i\bar{\epsilon}^-\tilde{\mathbb{F}} + i\epsilon^-\mathbb{F} \\
\delta_Q \bar{\Gamma}_- &= i\epsilon^+\tilde{\mathbb{F}} + i\bar{\epsilon}^+\mathbb{F} \\
\delta_Q \Gamma_- &= -i\epsilon^+\mathbb{F} - i\bar{\epsilon}^+\tilde{\mathbb{F}} .
\end{aligned} \tag{3.9}$$

It is easy to verify that these transformation laws are compatible with the transformation laws for the field strengths (3.1).

3.2 The nonabelian semichiral vector multiplet

We now extend the previous discussion to the nonabelian case [5].

For a nonabelian gauge group, the covariant derivatives are given by (2.14) and the field-strengths by the anticommutators in (2.16). The nonabelian version of the gauge transformations in (2.9) is

$$g(\Lambda) e^{\mathbb{V}^L} = e^{i\bar{\Lambda}_L} e^{\mathbb{V}^L} e^{-i\Lambda_L}$$

$$\begin{aligned}
g(\Lambda) e^{\mathbb{V}^R} &= e^{i\bar{\Lambda}_R} e^{\mathbb{V}^R} e^{-i\Lambda_R} \\
g(\Lambda) e^{\mathbb{V}_\phi} &= e^{i\Lambda_R} e^{\mathbb{V}_\phi} e^{-i\Lambda_L} \\
g(\Lambda) e^{\mathbb{V}_\chi} &= e^{i\bar{\Lambda}_R} e^{\mathbb{V}_\chi} e^{-i\Lambda_L} .
\end{aligned} \tag{3.10}$$

The nonabelian extensions of (2.11) and (2.12) are

$$\begin{aligned}
e^{\mathbb{V}^R} &= e^{\mathbb{V}_\chi} e^{-\mathbb{V}_\phi} = e^{-\bar{\mathbb{V}}_\phi} e^{\bar{\mathbb{V}}_\chi} \\
e^{\mathbb{V}^L} &= e^{\bar{\mathbb{V}}_\chi} e^{\mathbb{V}_\phi} = e^{\bar{\mathbb{V}}_\phi} e^{\mathbb{V}_\chi} .
\end{aligned} \tag{3.11}$$

Real representation

Finding additional supersymmetries for the semichiral multiplet is facilitated by working in a real representation. Below we present the material needed for this.

We introduce \mathbb{U}_L and \mathbb{U}_R such that

$$\begin{aligned}
e^{\mathbb{V}^L} &= e^{\bar{\mathbb{U}}_L} e^{\mathbb{U}_L} , \quad g(\Lambda, K) e^{\mathbb{U}_L} = e^{iK} e^{\mathbb{U}_L} e^{-i\Lambda_L} \\
e^{\mathbb{V}^R} &= e^{\bar{\mathbb{U}}_R} e^{\mathbb{U}_R} , \quad g(\Lambda, K) e^{\mathbb{U}_R} = e^{iK} e^{\mathbb{U}_R} e^{-i\Lambda_R} ,
\end{aligned} \tag{3.12}$$

with the parameter K an arbitrary Lie algebra valued gauge parameter, and thus

$$\begin{aligned}
e^{\mathbb{V}_\chi} &= e^{\bar{\mathbb{U}}_R} e^{\mathbb{U}_L} \\
e^{\mathbb{V}_\phi} &= e^{-\mathbb{U}_R} e^{\mathbb{U}_L} .
\end{aligned} \tag{3.13}$$

In the real representation, matter transforms as $M \rightarrow e^{iK} M$ and the derivatives read

$$\begin{aligned}
\bar{\nabla}_+ &= e^{\mathbb{U}_L} \bar{\mathbb{D}}_+ e^{-\mathbb{U}_L} \\
\nabla_+ &= e^{-\bar{\mathbb{U}}_L} \mathbb{D}_+ e^{\bar{\mathbb{U}}_L} \\
\bar{\nabla}_- &= e^{\mathbb{U}_R} \bar{\mathbb{D}}_- e^{-\mathbb{U}_R} \\
\nabla_- &= e^{-\bar{\mathbb{U}}_R} \mathbb{D}_- e^{\bar{\mathbb{U}}_R} .
\end{aligned} \tag{3.14}$$

The gauge transformations are

$$\begin{aligned}
e^{\mathbb{U}_L} \delta_g e^{-\mathbb{U}_L} &= -iK + e^{\mathbb{U}_L} i\Lambda_L e^{-\mathbb{U}_L} \\
e^{-\bar{\mathbb{U}}_L} \delta_g e^{\bar{\mathbb{U}}_L} &= -iK + e^{-\bar{\mathbb{U}}_L} i\bar{\Lambda}_L e^{\bar{\mathbb{U}}_L} \\
e^{\mathbb{U}_R} \delta_g e^{-\mathbb{U}_R} &= -iK + e^{\mathbb{U}_R} i\Lambda_R e^{-\mathbb{U}_R} \\
e^{-\bar{\mathbb{U}}_R} \delta_g e^{\bar{\mathbb{U}}_R} &= -iK + e^{-\bar{\mathbb{U}}_R} i\bar{\Lambda}_R e^{\bar{\mathbb{U}}_R} .
\end{aligned} \tag{3.15}$$

A transformation on Γ can be written as

$$\delta\Gamma = [\nabla, e^{-V} \delta e^V] , \tag{3.16}$$

hence, *e.g.*,

$$\delta_g \Gamma_+ = -i \nabla_+ K . \quad (3.17)$$

In the abelian case, $\delta \mathbb{V}^L = \delta \mathbb{U}_L + \delta \bar{\mathbb{U}}_L$, from which we derive the abelian supersymmetry transformations for \mathbb{U}_L ,

$$\delta_Q \mathbb{U}_L = \bar{\epsilon}^- \bar{\mathbb{D}}_- (\mathbb{V}_\phi + \bar{\mathbb{V}}_\chi) . \quad (3.18)$$

$N = (4, 4)$ supersymmetry

We assume that the supersymmetry transformation laws for the field-strengths (3.1) generalize:

$$\begin{aligned} \delta_Q \mathbb{F} &= i(\bar{\nabla}_+ \delta_Q \bar{\Gamma}_- + \bar{\nabla}_- \delta_Q \bar{\Gamma}_+) = \epsilon^- \bar{\nabla}_- \tilde{\mathbb{F}} + \epsilon^+ \bar{\nabla}_+ \tilde{\bar{\mathbb{F}}} \\ \delta_Q \tilde{\mathbb{F}} &= i(\bar{\nabla}_+ \delta_Q \Gamma_- + \nabla_- \delta_Q \bar{\Gamma}_+) = -\epsilon^+ \bar{\nabla}_+ \bar{\mathbb{F}} - \bar{\epsilon}^- \nabla_- \mathbb{F} . \end{aligned} \quad (3.19)$$

For a covariant derivative of the form

$$\nabla = D + \Gamma = e^{-V} D e^V , \quad (3.20)$$

an arbitrary variation can be written as in (3.16) and hence, *e.g.*,

$$\delta \bar{\Gamma}_+ = [\bar{\nabla}_+, e^{\mathbb{U}_L} \delta(e^{-\mathbb{U}_L})] . \quad (3.21)$$

For a supersymmetry transformation, starting from (3.9), we thus have

$$\begin{aligned} \delta_Q \bar{\Gamma}_+ &= i\epsilon^- \tilde{\bar{\mathbb{F}}} - i\bar{\epsilon}^- \mathbb{F} \\ &= \bar{\epsilon}^- \{\bar{\nabla}_+, \bar{\nabla}_-\} - \epsilon^- \{\bar{\nabla}_+, \nabla_-\} \\ &= \bar{\epsilon}^- \{e^{\mathbb{U}_L} \bar{\mathbb{D}}_+ e^{-\mathbb{U}_L}, \bar{\nabla}_-\} - \epsilon^- \{e^{\mathbb{U}_L} \bar{\mathbb{D}}_+ e^{-\mathbb{U}_L}, \nabla_-\} \\ &= e^{\mathbb{U}_L} (\bar{\epsilon}^- \{\bar{\mathbb{D}}_+, e^{-\mathbb{U}_L} \bar{\nabla}_- e^{\mathbb{U}_L}\} - \epsilon^- \{\bar{\mathbb{D}}_+, e^{-\mathbb{U}_L} \nabla_- e^{\mathbb{U}_L}\}) e^{-\mathbb{U}_L} \\ &= e^{\mathbb{U}_L} (\bar{\epsilon}^- \{\bar{\mathbb{D}}_+, e^{-\mathbb{U}_L} [\bar{\nabla}_-, e^{\mathbb{U}_L}]\} - \epsilon^- \{\bar{\mathbb{D}}_+, e^{-\mathbb{U}_L} [\nabla_-, e^{\mathbb{U}_L}]\}) e^{-\mathbb{U}_L} \\ &= \bar{\epsilon}^- \{\bar{\nabla}_+, [\bar{\nabla}_-, e^{\mathbb{U}_L}] e^{-\mathbb{U}_L}\} - \epsilon^- \{\bar{\nabla}_+, [\nabla_-, e^{\mathbb{U}_L}] e^{-\mathbb{U}_L}\} . \end{aligned} \quad (3.22)$$

Identifying this with (3.21) we find the supersymmetry transformations for the potentials $\mathbb{U}_{L,R}$,

$$\begin{aligned} e^{\mathbb{U}_L} \delta_Q(e^{-\mathbb{U}_L}) &= \epsilon^- (\nabla_- e^{\mathbb{U}_L}) e^{-\mathbb{U}_L} - \bar{\epsilon}^- (\bar{\nabla}_- e^{\mathbb{U}_L}) e^{-\mathbb{U}_L} \\ e^{\mathbb{U}_R} \delta_Q(e^{-\mathbb{U}_R}) &= \epsilon^+ (\nabla_+ e^{\mathbb{U}_R}) e^{-\mathbb{U}_R} - \bar{\epsilon}^+ (\bar{\nabla}_+ e^{\mathbb{U}_R}) e^{-\mathbb{U}_R} \\ e^{-\bar{\mathbb{U}}_L} \delta_Q(e^{\bar{\mathbb{U}}_L}) &= \bar{\epsilon}^- (\bar{\nabla}_- e^{-\bar{\mathbb{U}}_L}) e^{\bar{\mathbb{U}}_L} - \epsilon^- (\nabla_- e^{-\bar{\mathbb{U}}_L}) e^{\bar{\mathbb{U}}_L} \\ e^{-\bar{\mathbb{U}}_R} \delta_Q(e^{\bar{\mathbb{U}}_R}) &= \bar{\epsilon}^+ (\bar{\nabla}_+ e^{-\bar{\mathbb{U}}_R}) e^{\bar{\mathbb{U}}_R} - \epsilon^+ (\nabla_+ e^{-\bar{\mathbb{U}}_R}) e^{\bar{\mathbb{U}}_R} , \end{aligned} \quad (3.23)$$

which can be rewritten as

$$\begin{aligned}
(\delta_Q e^{-\mathbb{U}_L}) e^{\mathbb{U}_L} &= \epsilon^- e^{-\mathbb{V}_\chi} \mathbb{D}_- e^{\mathbb{V}_\chi} - \bar{\epsilon}^- e^{-\mathbb{V}_\phi} \bar{\mathbb{D}}_- e^{\mathbb{V}_\phi} \\
(\delta_Q e^{-\mathbb{U}_R}) e^{\mathbb{U}_R} &= \epsilon^+ e^{-\mathbb{V}_\chi} \mathbb{D}_+ e^{\mathbb{V}_\chi} - \bar{\epsilon}^+ e^{\mathbb{V}_\phi} \bar{\mathbb{D}}_+ e^{-\mathbb{V}_\phi} \\
(\delta_Q e^{\bar{\mathbb{U}}_L}) e^{-\bar{\mathbb{U}}_L} &= \bar{\epsilon}^- e^{\mathbb{V}_\chi} \bar{\mathbb{D}}_- e^{-\mathbb{V}_\chi} - \epsilon^- e^{\mathbb{V}_\phi} \mathbb{D}_- e^{-\mathbb{V}_\phi} \\
(\delta_Q e^{\bar{\mathbb{U}}_R}) e^{-\bar{\mathbb{U}}_R} &= \bar{\epsilon}^+ e^{\mathbb{V}_\chi} \bar{\mathbb{D}}_+ e^{-\mathbb{V}_\chi} - \epsilon^+ e^{-\mathbb{V}_\phi} \mathbb{D}_+ e^{\mathbb{V}_\phi} .
\end{aligned} \tag{3.24}$$

It is now straightforward to calculate the nonabelian supersymmetry transformations for the potentials \mathbb{V}_ϕ and \mathbb{V}_χ ,

$$\begin{aligned}
e^{-\mathbb{V}_\phi} (\delta_Q e^{\mathbb{V}_\phi}) &= e^{-\mathbb{U}_L} e^{\mathbb{U}_R} (\delta_Q e^{-\mathbb{U}_R}) e^{\mathbb{U}_L} + e^{-\mathbb{U}_L} (\delta_Q e^{\mathbb{U}_L}) \\
&= e^{-\mathbb{V}_\phi} (\delta_Q e^{-\mathbb{U}_R}) e^{\mathbb{U}_R} e^{\mathbb{V}_\phi} - (\delta_Q e^{-\mathbb{U}_L}) e^{\mathbb{U}_L} \\
&= \epsilon^+ e^{-\mathbb{V}_\phi} e^{-\mathbb{V}_\chi} (\mathbb{D}_+ e^{\mathbb{V}_\chi}) e^{\mathbb{V}_\phi} - \bar{\epsilon}^+ (\bar{\mathbb{D}}_+ e^{-\mathbb{V}_\phi}) e^{\mathbb{V}_\phi} - \epsilon^- e^{-\mathbb{V}_\chi} \mathbb{D}_- e^{\mathbb{V}_\chi} + \bar{\epsilon}^- e^{-\mathbb{V}_\phi} \bar{\mathbb{D}}_- e^{\mathbb{V}_\phi}
\end{aligned} \tag{3.25}$$

and

$$\begin{aligned}
e^{-\mathbb{V}_\chi} (\delta_Q e^{\mathbb{V}_\chi}) &= e^{-\mathbb{U}_L} e^{-\bar{\mathbb{U}}_R} (\delta_Q e^{\bar{\mathbb{U}}_R}) e^{\mathbb{U}_L} + e^{-\mathbb{U}_L} (\delta_Q e^{\mathbb{U}_L}) \\
&= e^{-\mathbb{V}_\chi} (\delta_Q e^{\bar{\mathbb{U}}_R}) e^{-\bar{\mathbb{U}}_R} e^{\mathbb{V}_\chi} - (\delta_Q e^{-\mathbb{U}_L}) e^{\mathbb{U}_L} \\
&= \bar{\epsilon}^+ (\bar{\mathbb{D}}_+ e^{-\mathbb{V}_\chi}) e^{\mathbb{V}_\chi} - \epsilon^+ e^{-\mathbb{V}_\chi} e^{-\mathbb{V}_\phi} (\mathbb{D}_+ e^{\mathbb{V}_\phi}) e^{\mathbb{V}_\chi} - \epsilon^- e^{-\mathbb{V}_\chi} \mathbb{D}_- e^{\mathbb{V}_\chi} + \bar{\epsilon}^- e^{-\mathbb{V}_\phi} \bar{\mathbb{D}}_- e^{\mathbb{V}_\phi} .
\end{aligned} \tag{3.26}$$

Closure of the algebra for the field-strengths

Unlike in the abelian case, where the field-strengths are invariant under gauge transformations, the gauge transformations in the closure of the supersymmetry algebra here also transform the field-strengths. Defining

$$\{\bar{\nabla}_+, \nabla_+\} = i\nabla_{++} = i\partial_{++} + i\Gamma_{++} , \tag{3.27}$$

for \mathbb{F} we find

$$\begin{aligned}
[\delta_1, \delta_2] \mathbb{F} &= \delta_1 (\epsilon_2^+ \bar{\nabla}_+ \tilde{\mathbb{F}} + \epsilon_2^- \bar{\nabla}_- \tilde{\mathbb{F}}) - (1 \leftrightarrow 2) \\
&= \epsilon_2^+ \bar{\nabla}_+ (-\bar{\epsilon}_1^+ \nabla_+ \mathbb{F} + \epsilon_1^- \bar{\nabla}_- \mathbb{F}) + \epsilon_2^- \bar{\nabla}_- (\epsilon_1^+ \bar{\nabla}_+ \mathbb{F} - \bar{\epsilon}_1^- \nabla_- \mathbb{F}) \\
&\quad + \epsilon_2^+ (i\epsilon_1^- [\tilde{\mathbb{F}}, \tilde{\mathbb{F}}] - i\bar{\epsilon}_1^- [\mathbb{F}, \tilde{\mathbb{F}}]) + \epsilon_2^- (i\epsilon_1^+ [\tilde{\mathbb{F}}, \tilde{\mathbb{F}}] + i\bar{\epsilon}_1^+ [\mathbb{F}, \tilde{\mathbb{F}}]) \\
&= \xi^{++} [\nabla_{++}, \mathbb{F}] + \xi^{--} [\nabla_{--}, \mathbb{F}] + \bar{\alpha} [\bar{\mathbb{F}}, \mathbb{F}] - \tilde{\alpha} [\tilde{\mathbb{F}}, \mathbb{F}] + \tilde{\tilde{\alpha}} [\tilde{\mathbb{F}}, \mathbb{F}] .
\end{aligned} \tag{3.28}$$

Generally, for any $F \in \{\mathbb{F}, \bar{\mathbb{F}}, \tilde{\mathbb{F}}, \tilde{\tilde{\mathbb{F}}}\}$, the supersymmetry algebra closes up to a gauge transformation,

$$[\delta_1, \delta_2] F = \xi^{++} \partial_{++} F + \xi^{--} \partial_{--} F + [K(\epsilon_1, \epsilon_2), F] , \tag{3.29}$$

with the hermitian gauge parameter $K(\epsilon_1, \epsilon_2)$ defined as

$$K(\epsilon_1, \epsilon_2) = \xi^+ \Gamma_+ + \xi^- \Gamma_- - \alpha \mathbb{F} + \bar{\alpha} \bar{\mathbb{F}} - \tilde{\alpha} \tilde{\mathbb{F}} + \tilde{\bar{\alpha}} \tilde{\bar{\mathbb{F}}} . \quad (3.30)$$

The semichiral parameters of eq. (3.6) generalize to

$$\begin{aligned} \Lambda_L &= -i \xi^+ \bar{\nabla}_+ e^{-\mathbb{V}^L} \mathbb{D}_+ e^{\mathbb{V}^L} , \\ \Lambda_R &= -i \xi^- \bar{\nabla}_- e^{-\mathbb{V}^R} \mathbb{D}_- e^{\mathbb{V}^R} . \end{aligned} \quad (3.31)$$

4 No $N = (4, 4)$ susy for the Large Vector Multiplet

We now turn to the Large Vector Multiplet which gauges isometries acting simultaneously on chiral and twisted chiral superfields, as described in section 2. As we shall see, the situation is completely different as compared to the semichiral multiplet discussed in the previous section. For the Large Vector Multiplet (LVM), the field-strengths are semichiral fermionic superfields. This fact implies that there is no $N = (4, 4)$ off-shell supersymmetry in the abelian case [1, 15].

The field-strengths for the abelian LVM defined in (2.21) are four semichiral fermions,

$$\mathbb{G}_+ = \bar{\mathbb{D}}_+ V_L , \quad \bar{\mathbb{G}}_+ = \mathbb{D}_- \bar{V}_L , \quad \mathbb{G}_- = \bar{\mathbb{D}}_- V_R , \quad \bar{\mathbb{G}}_- = \mathbb{D}_- \bar{V}_R . \quad (4.1)$$

As shown in paper [15] and discussed further in [1], additional off-shell $N = (4, 4)$ supersymmetry can only be imposed on a set of semichiral fields if the dimension of the space of fields is larger than four. It is therefore impossible to impose additional supersymmetry on the field-strengths of the abelian Large Vector Multiplet.

Left/right supersymmetry

Even though we cannot impose ordinary $N = (4, 4)$ supersymmetry, we could make an ansatz for independent *left* susy for \mathbb{G}_+ and *right* susy for \mathbb{G}_- as

$$\begin{aligned} \delta_Q \mathbb{G}_+ &= \bar{\epsilon}^- \bar{\mathbb{D}}_- \mathbb{G}_+ - \epsilon^- \mathbb{D}_- \mathbb{G}_+ \\ \delta_Q \mathbb{G}_- &= \bar{\epsilon}^+ \bar{\mathbb{D}}_+ \mathbb{G}_- - \epsilon^+ \mathbb{D}_+ \mathbb{G}_- , \end{aligned} \quad (4.2)$$

which closes to

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] \mathbb{G}_+ = \xi^- \partial_- \mathbb{G}_+ , \quad [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] \mathbb{G}_- = \xi^+ \partial_+ \mathbb{G}_- . \quad (4.3)$$

Twisted supersymmetry

Another option is to impose additional *pseudo*-supersymmetry. An ansatz for the semichiral field-strengths of the Large Vector Multiplet is

$$\begin{aligned}
\delta\mathbb{G}_+ &= \bar{\epsilon}^-\bar{\mathbb{D}}_-\mathbb{G}_+ + \epsilon^-\mathbb{D}_-\mathbb{G}_+ + \epsilon^+\bar{\mathbb{D}}_+\bar{\mathbb{G}}_+ \\
\delta\bar{\mathbb{G}}_+ &= \epsilon^-\mathbb{D}_-\bar{\mathbb{G}}_+ + \bar{\epsilon}^-\bar{\mathbb{D}}_-\bar{\mathbb{G}}_+ + \bar{\epsilon}^+\mathbb{D}_+\mathbb{G}_+ \\
\delta\mathbb{G}_- &= \bar{\epsilon}^+\bar{\mathbb{D}}_+\mathbb{G}_- + \epsilon^+\mathbb{D}_+\mathbb{G}_- + \epsilon^-\bar{\mathbb{D}}_-\bar{\mathbb{G}}_- \\
\delta\bar{\mathbb{G}}_- &= \epsilon^+\mathbb{D}_+\bar{\mathbb{G}}_- + \bar{\epsilon}^+\bar{\mathbb{D}}_+\bar{\mathbb{G}}_- + \bar{\epsilon}^-\mathbb{D}_-\mathbb{G}_- .
\end{aligned} \tag{4.4}$$

This ansatz closes to a pseudo-supersymmetry algebra

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\mathbb{G}_+ = -(\xi^+\partial_+\mathbb{G}_+ + \xi^-\partial_-\mathbb{G}_+). \tag{4.5}$$

Identifying the field-strength potential $\mathbb{G}_+ = \bar{\mathbb{D}}_+V_L$ and $\mathbb{G}_- = \bar{\mathbb{D}}_-V_R$ and using the reality constraint $V_L + \bar{V}_L = V_R + \bar{V}_R$, we find

$$\begin{aligned}
\delta V_L &= \bar{\epsilon}^-\bar{\mathbb{D}}_-V_L + \epsilon^-\mathbb{D}_-V_L - \epsilon^+\bar{\mathbb{D}}_+\bar{V}_L + \bar{\epsilon}^+\bar{\mathbb{D}}_+(2V_L + \bar{V}_L) , \\
\delta V_R &= \bar{\epsilon}^+\bar{\mathbb{D}}_+V_R + \epsilon^+\mathbb{D}_+V_R - \epsilon^-\mathbb{D}_-\bar{V}_R + \bar{\epsilon}^-\bar{\mathbb{D}}_-(2V_R + \bar{V}_R) .
\end{aligned} \tag{4.6}$$

The transformations close to a pseudo-supersymmetry algebra.

In [15], we show that $N = (4, 4)$ supersymmetry can be imposed for a set of semichiral fields if their number is $4d$ with $d > 1$. This might suggest that it is possible to find extra supersymmetries for the nonabelian Large Vector Multiplet. However, for a gauge multiplet the extra supersymmetry should commute with the gauge transformations. We do not believe that this is possible, at least not for an off-shell supersymmetry algebra. There are reasons from T-duality to believe that it might be possible to find an on-shell $(4, 4)$ -algebra for certain actions, but we have not investigated that option in detail.

5 Conclusions

In this paper, we discuss $(4, 4)$ supersymmetry for two gauge multiplets introduced in [5] and [6]; the semichiral multiplet and the Large Vector Multiplet.

For the semichiral vector multiplet, we find off-shell $(4, 4)$ supersymmetry, first for a $U(1)$ and then for a general gauge group. The transformations for the potentials are deduced from those of the field-strengths and close to a supersymmetry up to gauge transformations. In [19] a related multiplet was recently constructed in bi-projective superspace

[20]. We believe that a suitable partial gauge-fixing relates the two, but have not checked this.

For the Large Vector Multiplet we do not find off-shell $(4, 4)$ supersymmetry but are able to construct twisted $(4, 4)$ supersymmetry in the abelian case. This is in agreement with the results in [1, 15], where it was shown that $(4, 4)$ supersymmetry can be imposed on a set of semichiral fields parametrizing a d -dimensional target space only if $d > 4$.

Acknowledgements:

The authors wish to thank Warren Siegel for discussions. IR is happy to thank Uppsala University for hospitality. The research of UL was supported by VR grant 621-2009-4066 and that of MR was supported in part by NSF grant no. PHY-06-53342. The stimulating atmosphere at the 2010 Simon's summer workshop helped us to finish this paper.

A Hermiticity conventions

We are using the conventions of [18]. Briefly, this implies that a spinor with an upper index is hermitian as is a covariant derivative with a lower index:

$$(\psi^\pm)^\dagger = \bar{\psi}^\pm, \quad (\mathbb{D}_\pm)^\dagger = \bar{\mathbb{D}}_\pm. \quad (\text{A.1})$$

This implies that for an operator,

$$(\epsilon^\pm \mathbb{D}_\pm)^\dagger = -\bar{\epsilon}^\pm \bar{\mathbb{D}}_\pm, \quad (\text{A.2})$$

which mimics the relation for a bosonic derivative and parameter $(\xi^a \partial_a)^\dagger = -\bar{\xi}^a \partial_a$. The latter is the familiar fact that as an operator the bosonic derivative ∂_a is antihermitian. Nevertheless, of course $\partial_a f$ is real for a real function f . We shall find a similar situation in the fermionic case.

In [18] the infinitesimal transformation of a superfield ϕ is defined via a commutator,

$$\delta\phi = [\epsilon^\pm \mathbb{D}_\pm, \phi]. \quad (\text{A.3})$$

Using the relation (A.1) this implies that

$$\delta\bar{\phi} = (\delta\phi)^\dagger = [\epsilon^\pm \mathbb{D}_\pm, \phi]^\dagger = [\bar{\epsilon}^\pm \bar{\mathbb{D}}_\pm, \bar{\phi}]. \quad (\text{A.4})$$

Assuming that $\epsilon^\pm \mathbb{D}_\pm$ acts on ψ via a commutator, the relations (A.3) and (A.4) are written

$$\delta\phi = \epsilon^\pm \mathbb{D}_\pm \phi, \quad \delta\bar{\phi} = \bar{\epsilon}^\pm \bar{\mathbb{D}}_\pm \bar{\phi}. \quad (\text{A.5})$$

There is another way of arriving at the expressions in (A.5) that avoids explicitly considering the adjoint of operators. Using

$$D_\theta \theta = 1 \ , \tag{A.6}$$

and considering a constant anticommuting spinor η , we have the relation

$$\epsilon D_\theta \theta \eta = \epsilon \eta \ . \tag{A.7}$$

Conjugating this and remembering that $(\epsilon \eta)^\dagger = \bar{\eta} \bar{\epsilon} = -\bar{\epsilon} \bar{\eta}$ we find

$$(\epsilon D_\theta)^\dagger (-\bar{\theta} \bar{\eta}) = -\bar{\epsilon} \bar{\eta} \ , \tag{A.8}$$

from which it follows that $(\epsilon D_\theta \phi)^\dagger = \bar{\epsilon} \bar{D}_\theta \bar{\phi}$ for a superfield ϕ , and we recover (A.5) without involving commutators of operators.

References

- [1] M. Götteman, U. Lindström, M. Roček and I. Ryb, “Sigma models with off-shell N=(4,4) supersymmetry and noncommuting complex structures,” arXiv:0912.4724 [hep-th].
- [2] T. Buscher, U. Lindström and M. Roček, “New Supersymmetric σ -models with Wess-Zumino terms,” Phys. Lett. B **202**, 94 (1988).
- [3] S. J. Gates, C. M. Hull and M. Roček, “Twisted Multiplets And New Supersymmetric Nonlinear Sigma Models,” Nucl. Phys. **B248** (1984) 157.
- [4] N. J. Hitchin, A. Karlhede, U. Lindström and M. Roček, “Hyperkähler Metrics And Supersymmetry,” Commun. Math. Phys. **108**, 535 (1987).
- [5] U. Lindström, M. Roček, I. Ryb, R. von Unge and M. Zabzine, “New N = (2, 2) vector multiplets,” arXiv:0705.3201 [hep-th].
- [6] U. Lindström, M. Roček, I. Ryb, R. von Unge and M. Zabzine, “Nonabelian Generalized Gauge Multiplets,” arXiv:0808.1535 [hep-th].
- [7] U. Lindström, M. Roček, I. Ryb, R. von Unge and M. Zabzine, “T-duality and Generalized Kähler Geometry,” arXiv:0707.1696 [hep-th].

- [8] S. J. J. Gates and W. Merrell, “D=2 N=(2,2) Semi Chiral Vector Multiplet,” JHEP **0710**, 035 (2007) arXiv:0705.3207 [hep-th].
- [9] W. Merrell and D. Vaman, “T-duality, quotients and generalized Kähler geometry,” Phys. Lett. B **665**, 401 (2008) arXiv:0707.1697 [hep-th].
- [10] U. Lindström, M. Roček, R. von Unge and M. Zabzine, “Generalized Kaehler manifolds and off-shell supersymmetry,” Commun. Math. Phys. **269** (2007) 833 [arXiv:hep-th/0512164].
- [11] M. Gualtieri, “Generalized complex geometry,” Oxford University DPhil thesis, [arXiv:math/0401221].
- [12] M. T. Grisaru, M. Massar, A. Sevrin and J. Troost, “Some aspects of N = (2,2), D = 2 supersymmetry,” Fortsch. Phys. **47**, 301 (1999) [arXiv:hep-th/9801080].
- [13] S. J. Gates, “Why are there so many N=4 superstrings?,” Phys. Lett. B **338**, 31 (1994) [arXiv:hep-th/9410149].
- [14] M. Göteman, U. Lindström, W. Merrell and M. Roček, In preparation.
- [15] M. Göteman and U. Lindström, “Pseudo-hyperkähler Geometry and Generalized Kähler Geometry,” arXiv:0903.2376 [hep-th].
- [16] W. Merrell, L. A. P. Zayas and D. Vaman, “Gauged (2,2) Sigma Models and Generalized Kähler Geometry,” JHEP **0712**, 039 (2007) [arXiv:hep-th/0610116].
- [17] I. Ryb, “The Large Vector Multiplet Action,” arXiv:0710.3208 [hep-th].
- [18] S. J. Gates, M. T. Grisaru, M. Roček and W. Siegel, “Superspace, or one thousand and one lessons in supersymmetry,” Front. Phys. **58**, 1 (1983) [arXiv:hep-th/0108200].
- [19] G. Tartaglino-Mazzucchelli, “On 2D N=(4,4) superspace supergravity,” arXiv:0912.5300 [hep-th].
- [20] U. Lindström, I. T. Ivanov and M. Roček, “New N=4 superfields and sigma models,” Phys. Lett. B **328**, 49 (1994) [arXiv:hep-th/9401091].